Fourier Series & Transform

Fourier Series

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_0 t}$$

$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t) e^{-j2\pi n f_0 t} dt$$

or

$$x(t) = a_0 \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi t}{L} + b_n \sin\left(\frac{n\pi t}{L}\right)\right)$$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(t)dt$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(t)\cos\frac{n\pi t}{L} dt$$

$$b_n = \frac{1}{L}\int_{L}^{L} f(t)\sin\frac{n\pi t}{L} dt$$

Fourier Transform: $X(f) = \int_{-\infty}^{\infty} x(t).e^{-j2\pi ft} dt$

Inverse Fourier Transform: $x(t) = \int_{-\infty}^{\infty} X(f).e^{j2\pi ft} df$

Discrete Fourier Transform $S\left(\frac{kf_s}{N}\right) = \sum_{n=0}^{N-1} s(nT).e^{-j\frac{2\pi kn}{N}}$

Properties

	Time Domain where: $x(t) = F^{-1}\{X(f)\}$	Frequency Domain where: $X(f) = F\{x(t)\}$
Linearity	$a.x(t)$	$a.X(f)$
Scaling	$x(a.t)$	$\frac{1}{\lvert a\rvert} X\left(\frac{f}{a}\right)$
Time Shifting	$x(t-t_0)$	$X(f)e^{-j2\pi ft_0}$
Frequency Shifting	$e^{j2\pi f_0 t}x(t)$	$X(f-f_0)$
Duality	$X(t)$	$x(-f)$
Differentiation	$\frac{d}{dt}x(t)$	$j2\pi fX(f)$
Integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{1}{j2\pi f}X(f) + \frac{X(0)}{2}\delta(f)$
Convolution	$x_1(t).x_2(t)$ $x_1(t) * x_2(t)$	$X_1(f) * X_2(f)$ $X_1(f).X_2(f)$
Convolution	$x(t)\cos(2\pi f_0 t)$ $x(t)\sin(2\pi f_0 t)$	$\frac{1}{2}[X(f+f_0)+X(f-f_0)]$ $\frac{1}{2}[X(f+f_0)-X(f-f_0)]$
Area	$\int_{-\infty}^{\infty} x(t)dt = X(0)$ $\int_{-\infty}^{\infty} X(f)df = x(0)$	

Common Fourier Transform Equivalents

Time Domain	Frequency Domain
$\delta(t)$	1
1	$\delta(f)$
$A\cos(2\pi f_0 t + \phi)$	$\frac{A}{2}\left[e^{j\phi}\delta(f + f_0) + e^{-j\phi}\delta(f - f_0)\right]$
$A\sin(2\pi f_0 t + \phi)$	$\frac{A}{2}\left[e^{j\phi}\delta(f + f_0) - e^{-j\phi}\delta(f - f_0)\right]$
$\delta(t - t_0)$	$e^{-2\pi f t_0}$
$e^{2\pi f_0 t}$	$\delta(f - f_0)$
$u(t)$	$\frac{\delta(f)}{2} + \frac{1}{j2\pi f}$
$u(t)e^{-a.t}$ (a > 0)	$\frac{1}{a + j2\pi f}$
$rect\left(\frac{t}{T}\right)$ where: $\left(rect(x) = \begin{cases} 1 & -0.5 < x < 0.5 \\ 0 & \lvert x\rvert > 0.5 \end{cases}\right)$	$Tsinc(f.T)$ Where: $\left(sinc(x) = \frac{\sin(x)}{x}\right)$
$Wsinc(W.t)$	$rect\left(\frac{f}{W}\right)$
$\cos(2\pi f_0 t)\left[rect\left(\frac{t}{T}\right)\right]$	$\frac{T}{2}\left[\text{sinc}(f - f_0)T + \text{sinc}(f + f_0)T\right]$
$\begin{cases} 1 - \frac{\lvert t\rvert}{T} & \text{for } \lvert t\rvert \le T \\ 0 & \text{for } \lvert t\rvert > T \end{cases}$	$T.\text{sinc}^2(f.T)$
$\sum_{k=-\infty}^{\infty}\delta(t - kT_0)$	$f_0\sum_{n=-\infty}^{\infty}\delta(f - nf_0)$